Why Are All Communist Countries Dictatorial?

C.Y. Cyrus Chu* and Meng-Yu Liang†

June 27, 2022

Abstract

Suppose that people in a capitalist society order policies mainly by comparing their wealth changes induced by various policies. We show that property prices play the role of *aligning* individual preferences, which naturally modifies the axiom of universal domain of the social decision function. Under reasonable and testable assumptions, we prove that this alignment effect eliminates the possibility of Condorcet Cycles (Condorcet 1785) and warrants the transitivity of the majority voting mechanism. Taking away private property will reinstitute Condorcet Cycles, returning the inescapable conclusion of a dictatorial social decision rule, as claimed by Arrow (1951). Our result provides partial support to the practices of voting eligibility tests implemented in many democratic countries.

1. Introduction

Freedom House has ranked the freedom status of most countries for decades. Table 1 summarizes a 2X2 classification of 160 countries in 1989 before the Berlin Wall collapsed. One dimension of the Table is the political regime: democratic or dictatorial; and the other dimension is the economic system: capitalism or communism/socialism. The communism or socialism classification may be slightly vague because many countries have adopted mixed systems.[1] Still, this relatively minor vagueness does not affect the clear message revealed in this Table: capitalist countries may be democratic or dictatorial, but all communist countries are dictatorial.

A natural and intriguing question then arises: Why is this? Since communism does not allow the ownership of private property, our question can be reformulated as follows: how would the ownership of private property affect the social decision

* Institute of Economics, Academia Sinica, Taipei, Taiwan, email:cychu@econ.sinica.edu.tw
† Institute of Economics, Academia Sinica, Taipei, Taiwan, email: myliang@econ.sinica.edu.tw

[1] The vagueness also arises for Nordic countries where the *social democracy* regime dictates that many sectors in health care, infrastructure, and public utilities should be publicly owned. However, if we consider a narrower domain of property, then the vagueness is reduced. For instance, if land is the only property to be considered, then there is no vagueness at all: lands are publicly owned only in communist countries. See Brandal et al. (2013) for the details.

mechanism that is to be formed?

The most common social aggregation rule in democratic countries is majority rule. In the context of Arrow's impossibility theorem, the main problem with the majority voting mechanism is that it may violate the transitivity axiom. It has been shown that, other than the various conditions referred to in Sen and Pattanaik (1969),[2] some individual preference configurations may easily generate a Condorcet Cycle (CC in short) in which no candidate is pairwise-preferred over all opponents. This is a disturbing result, because a CC suggests that for every policy chosen by the society, there is another policy that the majority strictly prefers. Furthermore, the theoretical indeterminacy of the outcomes of majority decision-making makes democratic decisions arbitrary and manipulable (McKelvey, 1976, 1979; Schofield 1978; and Riker 1980).

This paper shows how property prices in a capitalist society play the role of *aligning* individual preferences, which naturally weakens the universal domain axiom of the impossibility theorem. Under reasonable and testable assumptions, we prove that this alignment effect eliminates the possibility of CCs and reinstitutes the transitivity of the majority voting. We also provide conditions for the Condorcet winner to coincide with the socially efficient policy.

1.1 Communist Land Reforms in the Past Century

Before we analyze the differences between free capitalist societies (in which private property ownership is allowed and protected, and there are corresponding property markets) and communist societies (in which private property ownership is not permitted, and there is no corresponding property market), we first provide some background analysis of China and the USSR, which may give us some additional insights.

The Red Revolution in 1917 birthed the first communist country in the world. The central conflict described in Karl Marx's *Das Kapital* (2008) was between capitalists and laborers. However, since Russia was not very industrialized at the beginning of the 20th century, the "capitalist" class was not typical. Therefore the main class to be "emancipated" consisted not of laborers but of peasants. As such, the main means of production to be "socialized" was land (Kenez 1999).

In the USSR, the farmlands being expropriated were later collectively used, either in joint cultivation or as *artels* (agricultural communes). Land reform in the USSR was implemented mainly in the 1930s. The statistics show that between 1929 and 1940, the percentage of sown area in collective use increased from 4.9% to 99.8%. In short,

---

[2] The conditions include: extremal restrictions (ER), value restrictions (VR) and limited agreement (LA). However, these conditions are more like the results of mathematical reasonings and lack general economic implications.

within eleven years, nearly all privately-owned land was taken away (Fitzpatrick 1994).

The land reform process in China was even faster and harsher. By 1947 when the Chinese Communist Party took power, the economy was impoverished, and there were hardly any capitalists left to fight against. Because peasants were the only group that needed to be "emancipated," as in the USSR, Mao Zedong's priority policy was land reform. China's land reform started in 1953 and, within five years, private ownership of land was entirely eliminated. As in the USSR, the expropriated lands were organized into state-operated communes or agricultural production cooperatives, in which all factors of production, in particular land, were centrally controlled (DeMare 2019). The pace of Chinese land reform was much more rapid, mainly because it was more violent.[3]

To sum up, in both the USSR and China, there was initially private ownership of lands, and the communist regime abolished it in the name of land reform. After the reform, the real estate market essentially vanished. The main question to be addressed in this paper is: After the land property right was taken away and the market for land ceased to exist, how would individual preferences change? Furthermore, how would the social decision rule be different? For instance, if the government were to pick out a site to develop a park, how would the social decision be different, with or without the private ownership of land?[4]

1.2 Arrow's Social Decision Framework and Related Literature

One possible way of addressing the question mentioned above is to place it in the context of Arrow's social choice framework. Arrow's famous *Impossibility Theorem* states that, given some reasonable axioms, a nondictatorial social decision rule does not exist. In the past seventy years, there have been hundreds of articles that have tried to modify Arrow's result, showing that the conclusion of a dictatorial decision rule may be avoided if some axioms are dropped or weakened. One direction of such research that is related to our discussion is to modify the assumption of universal domain (UD in short), which states that the social decision rule should be applicable to all possible individual orderings. We will shortly explain how the individual property right is related to the UD assumption, but before that let us briefly review the

[3] It was estimated that millions of landlords and rich peasants were killed. The estimated number of deaths varies from 830,000 to 3,000,000, according to different sources. Evidence abounds. See https://en.wikipedia.org/wiki/Land_Reform_Movement_(China) and the references therein.

[4] One may argue that the mandatory expropriation of land itself is authoritarian, and hence it is not surprising that an authoritarian regime turns out to be dictatorial in policy making. While this argument may be logical, it does not distinguish the land expropriation *per se* and the social decision after land property rights are taken away. Theoretically and conceptually, even if all means of production are publicly owned, this does not imply that there is any particular form of social decision making. In *Das Kapital*, Karl Marx never connected the communist revolution with dictatorial public policy decision-making.

related literature.

Regarding the UD assumption, most researchers often refer to the following interpretation. Arrow (1951, p.24) stated that "(i)f we do not wish to require any prior knowledge of the tastes of individuals before specifying our social welfare function, that function will have to be defined for every logically possible set of individual orderings. Such a social welfare function would be universal in the sense that it would be applicable to any community." On the same page, Arrow also said that this UD assumption was based on our lack of knowledge about individual ordering: "the ordering by any individual is completely unknown in advance." One remark we should make here is that while it is true that we do not have knowledge about individual orderings, this does not mean that we have no idea about *what factors may affect such orderings*.

Several types of contribution have been made to revise the UD assumption. One line of research is related to individual preference orderings that are not allowed or are restricted in the presence of other people's preference relations (see Sen 1966; Sen and Pattanaik 1969). A particular example is a single-peak preference, which is known as a condition to sustain the transitivity of the majority-voting mechanism. Another line of research considers conditions regarding the distribution of preferences under which preference profiles can be reduced (Gaertner 1988). Some scholars have admitted the possible failure of having a nondictatorial social aggregation rule under some preference profiles, and have calculated the probability of successfully generating a transitive aggregation result (DeMeyer and Plott 1970). There is also a line of research that compares the majority rule with other social aggregation rules that fulfill the desirable transitivity property, and checks which rule corresponds to the broadest class of preference domain (Maskin 1995).

Of course our brief review above is not exhaustive; a more comprehensive literature survey can be found in the chapter by Gaertner (2002). To the best of our knowledge, however, no one has accomplished any analysis on the social decision rule with respect to whether or not private property is allowed, which is the key distinction between capitalism and communism. It is indeed important to study the relationship between social decision rules and private property ownership because, as pointed out by Sen (1970, p.165), "individual preferences are determined not by turning a roulette wheel over all possible alternatives, but by certain special social, economic, political, and cultural forces. This may easily produce some patterns in the set of individual preferences." Evidently, private property is arguably the most important economic force in a capitalist society. In the next section we will explain how private property rights may change individual orderings.

1.3 Summary of Results and the Structure of this Paper

This paper aims to examine how individual property rights affect the social aggregation rule. We will construct a model of a social decision framework in which private property ownership is allowed. To make our model mirror the scenarios in China and the USSR in the first half of the 20th century, we consider land as the only property that people own. Considering land as the sole form of property not only fits the historical background of the two largest communist countries in the world, but also has an analytical advantage: it helps us match the technical setting with the empirical relevance.

Our main point is the following: individual orderings depend on both their endowment specifications and their tastes. When all citizens' endowments are taken away, what differentiates individual preferences is just their respective tastes. Because we cannot rule out any kinds of individual tastes, Arrow's assumption of the universal domain is naturally persuasive in a society without private property, and therefore his impossibility theorem holds, leading to the inescapable conclusion of a dictatorial decision. In a capitalist society with individual property endowments, however, the prices in the property market provide a natural alignment of individual preferences so that their ordering of policy options is somewhat related, despite their endowments varying. This price alignment creates a particular type of *restricted domain*, which helps to lead to a nondictatorial social aggregation rule. The above description provides an answer to the question that we abstracted from the Freedom House survey: Why are all communist countries dictatorial, while capitalist countries may be democratic or dictatorial? Communist countries are dictatorial because 1) citizens' property rights are taken away, which highlights the idiosyncrasy of individual tastes, and 2) the alignment effect of the market price vanishes, which further relaxes the domain of individual preferences.

The remainder of this paper is arranged as follows. Section 2 presents a model of social choice function with or without private properties and explains the connection between private properties and the UD axiom. Section 3 demonstrates how the CC can be avoided in a society with a simple structure of property ownership. Section 4 extends the results in Section 3 to the case of property ownership portfolios. Section 5 discusses situations where there are more than three policy options or where there are concerns other than property ownership. The final section concludes.

2. Social Choice Function and the Condorcet Cycle

Let us briefly introduce some basic notations and definitions. The theory of social choices tries to aggregate individual orderings of policies into a social ordering.

Let the ordering of individual i be P_i. We write it as a function: $P_i = P_i(S; \omega_i, \theta_i)$, where *S* is the set of policies to be ordered, ω_i is the endowment of individual i, and θ_i is i's taste. A social choice function is a mapping from the product space $\prod_i P_i$ to the space of social ordering *P*, which we write as $\prod_i P_i \to P$. The reason why we specify P_i as a function of both ω_i and θ_i will be explained shortly.

2.1 Social Choice Functions with or without Private Property

As in the literature, we assume that individual preference P_i is complete and transitive for every i. As in Arrow (1951), we also adopt the following axioms: 1) the social ordering *P* should be complete and **transitive**, 2) $\prod_i P_i \to P$ should respect the **Pareto Principle** (if everyone prefers policy A to B, then the society should also prefer *A* to *B*), 3) $\prod_i P_i \to P$ should fulfill **IIA**, the independence of irrelevant alternatives (the social ordering of policy *A* versus policy *B* should not be affected by how people think of policy *C*), and 4) $\prod_i P_i \to P$ should work for the UD (**universal domain**, so that all points in $\prod_i P_i$ are possible).

We will now elaborate more on the meaning of the UD axiom. When we write individual i's ordering as $P_i = P_i(S; \omega_i, \theta_i)$, it means that i's preference over *S* is affected by both her endowment ω_i and her tastes θ_i. The UD assumption says that we may have all kinds of (ω_i, θ_i) for the possible P_i configurations. In terms of political philosophy, the UD assumption should be true behind the veil of ignorance (Rawls 1971), but its meaning in real-world operation needs to be clarified.

Arrow's theorem may be interpreted as a paradox of constitutional design behind the veil of ignorance. However, when we are out of the veil of ignorance and implement policy decisions, individual endowments ω_i are in fact known to each person i (and even possibly observable to other people). However, individual taste parameters θ_i are still unobservable. Thus, if we are to design a social decision rule that fulfills the assumption of UD, in a society allowing property ownership, we should expect that θ_i is not observable at all, but that ω_i is known to each individual i. If that is the situation, we may expect to reach some different conclusions, because people's decisions should be affected by their property endowments.

As we mentioned, endowment specification is the key difference between capitalism and communism: the strict definition of communism is that people do not have their private property or endowments, whereas in a capitalist society, people have a spectrum of endowments. A specified endowment is likely to restrict an individual's ordering of policy candidates, making the perturbation of preferences (which is needed to prove Arrow's theorem) inapplicable.

In the next section we demonstrate how the CC can be avoided in majority voting when there is a simple structure of property ownership. The remaining section explains why the CC exists when there is no private property.

2.2 Long vs. Short Condorcet Cycles

Consider two policies *A* and *B*. If the society prefers *A* to *B*, we write it as $A \succ B$. First, we will show below that, whenever there is a CC formed by more than three policies, there always exists a shorter cycle. Thus, without loss of generality, in most of our following discussion, we will consider CC to be formed by three policies: $A \succ B \succ \mathrm{C} \succ A$.

Suppose there is a four-policy cycle: *A*≻*B*, *B*≻*C*, *C*≻*D*, and *D*≻ A. Given *A*≻*B* and *B*≻*C*, we first ask whether it is true that *C*≻*A*. If *C*≻*A* holds, then we have a cycle among three policies *A*, *B* and *C*. If instead *A*≻ C, then we have *A*≻ C, *C*≻*D* and *D*≻*A*, which is another *A*-*C*-*D* three-policy cycle. The above argument can be extended to policy cycles $A_1 \succ A_2 \succ \cdots \succ A_n$ for any $n > 3$: For any n-policy CCs with n>3, there always exists a three-policy shorter cycle. Thus, our following discussion will mostly focus on three-policy cycles.

2.3 Condorcet Cycle in the Case Without Private Property

Consider three policy options *A*, *B*, and *C*. The policy in question is to develop a park on one of the three sites, denoted as *a, b* and *c,* respectively. Note that we use lower case letters (*a, b, c*) to denote sites and upper case letters (*A*, *B*, *C*) to denote policies. Let us first briefly describe what Arrow's theorem predicts in this park-developing social choice problem.

Suppose there are populations residing on these three sites, and that they prefer respectively that the park be built on a particular site. For instance, *a*-residents may prefer policy *A*, and between *B* and *C* there are two possibilities: either *B* is preferred or *C* is preferred. If an individual prefers *A* over *B* and *B* over *C*, to simplify the notation, we will express his or her ordering as *ABC*. There are six ordering possibilities, which we write below, together with their respective assumed population proportions:

$$ABC: N_1; \quad ACB: N_2; \quad CAB: N_3;$$
$$CBA: N_4; \quad BCA: N_5; \quad BAC: N_6.$$

Because the N_i's $(i = 1,2,\ldots,6)$ are population proportions; we have $\sum_i N_i = 1.$,

In this three-policy context, a CC will exist if the majority mechanism ends up with (*A*≻*B*, *B*≻*C* and *C*≻*A*), or (*B*≻ A, *C*≻*B* and *A*≻ C). In the former case, for instance, it means that

$$N_1 + N_2 + N_3 > N_4 + N_5 + N_6,$$
$$N_1 + N_5 + N_6 > N_2 + N_3 + N_4,$$
$$N_3 + N_4 + N_5 > N_1 + N_2 + N_6.$$

Given the axiom of universal domain, all $(N_1, N_2, N_3, N_4, N_5, N_6)$ vectors (and hence their preferences in the background) are possible as long as they sum up to 1. Thus, it is always possible for the above three inequalities to hold simultaneously,[5] and hence a CC may arise under some $(N_1, N_2, N_3, N_4, N_5, N_6)$ configurations.

3. How Do Property Ownership and Markets Eliminate Condorcet Cycles?

Now we bring in private property, a key feature in a capitalist society: residents at sites *a, b,* and *c* are assumed to own (real estate) properties on these sites. We first consider the simplest case: they only own properties on their respective residential sites, and nowhere else. The cases where they own multiple properties on other sites will be discussed later. We also assume that there is a property market that determines prices for each site, which we denote respectively as p_a, p_b and p_c.

3.1 Price Changes Caused by Policies

A pleasant park developed at a person's place of residence is supposed to raise the property price at that place, and the corresponding property prices at other sites may also appreciate or depreciate accordingly. With the private ownership of land, individual preferences over policies regarding park-site choices are no longer arbitrary; they usually prefer a policy that raises their property value more. We do not exclude other factors that may influence individual preferences, but a comparison of property values is without doubt the most reasonable baseline assumption that we should work with in a capitalist society. Note that with private property, the assumption of universal domain is related to the individual property ownership: people's preferences differ (partly or even mainly) because they have different property endowments.

Because a park can also be enjoyed by people traveling from other sites, a new park at site *a* will affect not only the property value at *a*, but also the property values at sites *b* and *c.* Furthermore, the same is true if the park is developed at site *b* (or *c*). When the park is developed at site *x*(*x*=*a, b, c*), we denote the percentage price changes at sites *a*, *b* and *c,* respectively, as $(\Delta p_{xa}, \Delta p_{xb}, \Delta p_{xc})$. Note that in the above, we use price change *percentages* rather than the absolute amount. This implies that there is no concern for absolute property values.

3.2 How Do Property Markets Eliminate CC?

First, we make a simplifying assumption.

Assumption 1 (A1) The price changes are symmetric: $\Delta \boldsymbol{p_{ab}} = \Delta \boldsymbol{p_{ba}}$, $\Delta \boldsymbol{p_{ac}} = \Delta \boldsymbol{p_{ca}}$, **and** $\Delta \boldsymbol{p_{bc}} = \Delta \boldsymbol{p_{cb}}$.

[5] For example, $N_1 = N_3 = N_5 = 0.2, N_2 = N_4 = 0.15$ and $N_6 = 0.1$.

That is, the price (percentage) change impact on site *x* when a park is developed at site *y* is the same as the price change impact on site *y* when a park is developed at site *x*. Intuitively, if the price change reveals the convenience of residents' traveling across places to enjoy the park, then A1 indicates that there is symmetric two-way traffic: the convenience for *x*-residents to enjoy a park at *y* is the same as that for *y*-residents to enjoy a park at *x*.

There are three between-site convenience possibilities among sites *a, b* and *c*. We denote the price appreciation (or depreciation) rates from the most convenient to the least convenient as α, β and γ, respectively. Recall that the price appreciation percentage at site *x* (*x*=*a, b, c*) is denoted by Δp_{xx}. Because an on-site visit to the park is most convenient, without loss of generality, we assume that $\gamma \leq \beta \leq \alpha \leq \Delta p_{xx}, x = a, b, c$. In Figure 1, we depict for demonstration purposes that α corresponds to the price-change parameter between sites *a* and *c*, β corresponds to the price-change parameter between sites *a* and *b*, and γ corresponds to the price-change parameter between sites *b* and *c*.[6] In terms of price changes, Figure 1 corresponds to

$$\alpha \equiv \Delta p_{ac} = \Delta p_{ca} \geq \beta \equiv \Delta p_{ab} = \Delta p_{ba} \geq \gamma \equiv \Delta p_{bc} = \Delta p_{cb}.$$

We normalize the total population in *a, b,* and *c* to be 1. Let the population at site x be $N_x, x = a, b, c$, and then we have $N_a + N_b + N_c = 1$. We consider a large population size, so that in most of our discussion we can ignore the event of a voting tie, which has a probability of nearly zero. In view of Figure 1, we know that, if people prefer policies that raise their property value more, then residents at site *a* must have preference *ACB* (because $\Delta p_{aa} > \Delta p_{ac} > \Delta p_{ab}$). Similarly, Figure 1 tells us that people at site *b* will have the preference *BAC*, and for residents at *c* they have the preference *CAB*.

Given the above property value-based preferences, if there is a CC formed by 1) $A \succ B$, $B \succ C$ and $C \succ A$, it must be the case that

$$A \succ B \Longrightarrow N_a + N_c > \frac{1}{2},$$

$$B \succ C \Longrightarrow N_b > \frac{1}{2},$$

$$C \succ A \Longrightarrow N_c > \frac{1}{2}.$$

However, this is impossible, because the first two inequalities together imply that $N_a + N_b + N_c > 1$, contradicting $N_a + N_b + N_c = 1$. Similarly, if we have a CC

[6] Note that the most convenient pair (between sites *a* and *c*) is the one having the largest price appreciation percentage α. However, in Figure 1, α corresponds to the smallest distance.

formed by 2) $B \succ A$, $C \succ B$ and $A \succ C$, it can be seen that there is also a contradiction. Because 1) and 2) are the only two CC types, we therefore have

Proposition 1: Suppose people form their orderings over policies by comparing the respective wealth changes. Under A1, if the price-change parameters $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ among *a, b* and *c* are such that $\boldsymbol{\alpha} > \boldsymbol{\beta} > \boldsymbol{\gamma}$ (as shown in Figure 1), then the wealth-based preferences cannot generate a CC.

Now, what if two of the three parameters α, β and γ are equal (see Figure 2)? There are two possibilities, respectively, in the two panels of Figure 2. Panel 2a refers to $\alpha = \beta$, and panel 2b refers to $\beta = \gamma$. In the case of panel 2a, residents at site *c* must have preference *CAB*, and residents at site *b* must have preference *BAC*. Residents at site *a* may have two preference types: some of them have *ABC* (assuming the proportion $\delta < 1$), and others have *ACB* (with the proportion $1 - \delta$). To have a CC formed by 1) $A \succ B$, $B \succ C$ and $C \succ A$, we must have

$$N_a + N_c > \frac{1}{2},$$

$$N_b + \delta N_a > \frac{1}{2},$$

$$N_c > \frac{1}{2}.$$

However, the last two inequalities together imply that $\delta N_a + N_b + N_c > 1$, which contradicts $N_a + N_b + N_c = 1$ (because $\delta < 1$). Similarly, one can show that a CC formed by 2) $B \succ A$, $C \succ B$ and $A \succ C$ also gives rise to a contradiction.

In the case of 2b, residents at *a* have *ACB,* and residents at *c* have *CAB*. Residents at *b* consist of two types: some have *BCA* (with proportion δ'), and others have *BAC*. To have a CC formed by 1) $A \succ B$, $B \succ C$ and $C \succ A$, we must have

$$N_a + N_c > \frac{1}{2},$$

$$N_b > \frac{1}{2},$$

$$\delta' N_b + N_c > \frac{1}{2}.$$

However, the first two inequalities together imply that $N_a + N_b + N_c > 1$, which contradicts $N_a + N_b + N_c = 1$. Similarly, one can show that a CC formed by 2) $B \succ A$, $C \succ B$ and $A \succ C$ also gives rise to a contradiction.

Summarizing the above, we have

Proposition 2: Suppose people form their preferences over policies by comparing the respective wealth changes. Under A1, if the convenience parameters among *a*, *b* and *c* are $\alpha = \beta$ or $\beta = \gamma$, then the wealth-based preferences cannot generate a CC.

3.2 The Alignment Effect of Private Property

What about the case where $\alpha = \beta = \gamma$? This is the case where the three possible sites are pairwise equally convenient. It can be shown that only in this case is a CC possible. Residents at *a* can have preferences *ABC* or *ACB*. Let the proportion of the former be δ_a. Similarly, residents at site *b* have preferences of either *BCA* (with the proportion δ_b) or *BAC*. Residents at site *c* have preferences of either *CAB* (with the proportion δ_c) or *CBA*. To have a CC with *A*≻*B*, *B*≻*C* and *C*≻*A*, we must have

$$N_a + \delta_c N_c > \frac{1}{2},$$

$$N_b + \delta_a N_a > \frac{1}{2},$$

$$N_c + \delta_b N_b > \frac{1}{2}.$$

Given the full degree of freedom of $(\delta_a, \delta_b, \delta_c)$, it is possible for the above three inequalities to hold simultaneously.

Summarizing the above, we see that other than the extreme case where $\alpha = \beta = \gamma$ (the traveling convenience is exactly the same between any two sites), a CC is never possible.

The intuition behind the above result is the following: when a policy is implemented, the property market in a capitalist society dictates price changes that *apply to all*. If people make decisions based on their wealth changes, the price changes play the role of "aligning" the domain of individual preferences. In Arrow's original setting, preferences over policies are determined by individual tastes, or "genes," whereas, in a capitalist society, preferences are very much determined by private property ownership and wealth changes. Note that genes cannot be "aligned," but wealth changes may be. To the best of our knowledge, the alignment effect of price changes in a capitalist society has not been analyzed in the literature.

In the special case where $\alpha = \beta = \gamma$, the price changes among the three sites are all the same ($\Delta p_{ab} = \Delta p_{ba} = \Delta p_{ac} = \Delta p_{ca} = \Delta p_{bc} = \Delta p_{cb}$). This eliminates the price alignment effect and the $(\delta_a, \delta_b, \delta_c)$ vector captures the pure taste effect, which therefore sustains the CC.

4. The General Case: People Own a Portfolio of Properties

Now we move to the more general property-ownership situation: people may own properties on multiple sites. Suppose a person's asset allocation proportion on sites (*a, b, c*) is (w_a, w_b, w_c), with $w_a + w_b + w_c = 1$. Thus the total wealth percentage change of this person for developing a park at site *x* is the inner product of her asset allocation proportion vector and the price-change vector, i.e., $(w_a, w_b, w_c) \cdot (\Delta p_{xa}, \Delta p_{xb}, \Delta p_{xc})$. Note that we define (w_a, w_b, w_c) as *percentages* of wealth instead of wealth values, so that people with different endowment sizes (rich and poor) face the same decision problem when they share the same (w_a, w_b, w_c). Thus, the possible conflicts between the rich and poor are not an issue in our analysis.

As before, we assume symmetric price-change parameters: $\Delta p_{ac} = \Delta p_{ca} = \alpha$, $\Delta p_{ab} = \Delta p_{ba} = \beta$, and $\Delta p_{bc} = \Delta p_{cb} = \gamma$, with $\alpha \geq \beta \geq \gamma$. For algebraic tractability, we further assume

Assumption 2 (A2): $\boldsymbol{\Delta p_{aa} = \Delta p_{bb} = \Delta p_{cc} = K}$**.**

A2 says that the price change percentages are the same for on-site park development.[7] Because an on-site visit is the most convenient one, we have $K > \alpha \geq \beta \geq \gamma$. Note that, although K is likely to be positive, α, or β, or γ may well be negative, meaning that, for instance, developing a park at site *a* depreciates the prices of properties at site *b* or *c*.

4.1 Separating Hyperplanes Formed by Property Price Changes

If a person is to compare her total wealth percentage change between two site choices *a* and *b*, he or she should check whether

$$(w_a, w_b, w_c) * (\Delta p_{ba} - \Delta p_{aa}, \Delta p_{bb} - \Delta p_{ab}, \Delta p_{bc} - \Delta p_{ac}) \quad (1)$$

is positive or negative.

Given A1 and A2, the wealth comparison in (1) shows that people would prefer policy *A* over policy *B* if their asset endowments satisfy

$$(\Delta p_{aa} - \Delta p_{ba})(w_a - w_b) + w_c(\Delta p_{ac} - \Delta p_{bc}) > 0.$$

Similarly, people would prefer *B* over *C* if their asset endowments satisfy

$$w_a(\Delta p_{ba} - \Delta p_{ca}) + (w_b - w_c)(\Delta p_{bb} - \Delta p_{cb}) > 0.$$

And they would prefer *C* over *A* if

$$(\Delta p_{cc} - \Delta p_{ac})(w_c - w_a) + w_b(\Delta p_{cb} - \Delta p_{ab}) > 0.$$

Using the property $(w_a + w_b + w_c) = 1$ to write w_c as $w_c = 1 - w_a - w_b$,

[7] If A2 does not hold, then Δp_{aa}, Δp_{bb} and Δp_{cc} may be different, and the algebra becomes rather cumbersome. This, however, does not affect the insight of our analysis. Note that since Δp_{aa}, Δp_{bb} and Δp_{cc} are defined as *percentage* changes, A2 does allow different appreciation *values* across places.

we can simplify the expressions of wealth changes. It can be shown that a person would prefer *A* over *B* if his or her asset endowments satisfy

$$(K-\alpha-\beta+\gamma)w_a+(\beta+\gamma-K-\alpha)w_b+(\alpha-\gamma)>0. \quad (2)$$

The hyperplane in Figure 3 marked $\overline{AB}$ corresponds to the case where (2) holds with equality. Similarly, *B* is preferred to *C* if

$$(K-\gamma+\beta-\alpha)w_a+2(K-\gamma)w_b-(K-\gamma)>0. \quad (3)$$

The hyperplane in Figure 3 marked $\overline{BC}$ corresponds to the case where (3) holds with equality. And *C* is preferred to *A* if

$$-2(K-\alpha)w_a+(\gamma-\beta-K+\alpha)w_b+(K-\alpha)>0. \quad (4)$$

The hyperplane in Figure 3 marked $\overline{CA}$ corresponds to the case where (4) holds with equality.

In the simplex $(w_a+w_b+w_c)=1$ (or $w_a+w_b\le 1$) in Figure 3, we identify six regions partitioned by the three hyperplanes $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$, with the population, respectively $(N_1,N_2,N_3,N_4,N_5,N_6)$.[8] We normalize the population to be $(N_1+N_2+N_3+N_4+N_5+N_6)=1$. To have a CC of the form *A≻B, B≻C, C≻A*, we need the following three inequalities to hold simultaneously:

$$(N_1+N_2+N_3)>(N_4+N_5+N_6), \quad \text{(5a)}$$
$$(N_1+N_5+N_6)>(N_2+N_3+N_4), \quad \text{(5b)}$$
$$(N_3+N_4+N_5)>(N_1+N_2+N_6). \quad \text{(5c)}$$

Note that we are back to the same situation as discussed in the case without private properties in Section 3.1. The question is: Can we show that CC like this is out of the question under some conditions on price-change parameters? We should emphasize that the conditions which we are looking for should not be related to the distribution of the population [characterized by the vector $(N_1,N_2,N_3,N_4,N_5,N_6)$], which may well be arbitrary in the spirit of universal domain.

4.2 Characteristics of the Hyperplanes and the Non-existence of CCs

We first study the intercepts and slopes of the three lines in Figure 3. For line $\overline{BC}$ corresponding to (3), one can easily identify the following features.

Facts for line $\overline{BC}$:

1) the intercept at $w_a=0$ is $w_b=1/2$,

2) its slope is negative:

$$\text{slope of } \overline{BC}=\frac{-[(K-\gamma)-(\alpha-\beta)]}{2(K-\gamma)}=\frac{-[(K-\alpha)+(\beta-\gamma)]}{2(K-\gamma)}<0.$$

3) the absolute value of this slope is less than 1/2.

For line $\overline{CA}$ corresponding to (4), we have

[8] Note that the sum of the LHS of (2), (3), and (4) is zero. Thus, the three lines $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ must intersect at the same point.

Facts for line $\overline{CA}$:

1) when $w_b = 0$, the intercept is $w_a = 1/2$.

2) Its slope is also negative:

$$\text{slope of } \overline{CA} = \frac{-2(K-\alpha)}{[(K-\alpha)+(\beta-\gamma)]} < 0.$$

3) If $K-\alpha < \beta-\gamma$, the absolute value of this slope is smaller than 1.

4) If $K-\alpha > \beta-\gamma$, then the absolute value of this slope is greater than 1 and less than 2.

For line $\overline{AB}$ corresponding to (2), we have found

Facts for line $\overline{AB}$:

1) When $w_a = 0$, its intercept is $w_b = (\alpha-\gamma)/(\alpha+K-\beta-\gamma) > 0$, which is greater (smaller) than 1/2 if $K-\alpha < (>) \beta-\gamma$.

2) Its slope is positive (negative) if $K-\alpha > (<) \beta-\gamma$:

$$\text{slope of } \overline{AB} = \frac{(K-\beta)-(\alpha-\gamma)}{(K-\beta)+(\alpha-\gamma)}.$$

3) When $K-\alpha < \beta-\gamma$, the absolute value of the slope is less than 1.

4) the line $\overline{AB}$ must go through the point $w_a = w_b = 1/2$.

We will separate our following discussion into two cases, depending on whether $K-\alpha$ is larger or smaller than $\beta-\gamma$.

4.3 The $K-\alpha < \beta-\gamma$ Case

The hyperplane information summarized in Section 4.2 allows us to draw Figure 4 for the case where $K-\alpha < \beta-\gamma$. In this case, we can see that the asset endowments corresponding to CBA and BCA disappear. Hence, $N_4 = N_5 = 0$. Suppose $C \succ A$. From 5(c) we need to have $N_3 > 1/2$ (the green area). Hence, the population with preference CAB always wins a majority of votes, i.e., we have a well-defined social ordering $P = CAB$. Similarly, we can show that neither is a CC of the form [$B \succ A, C \succ B$ and $A \succ C$] possible, because $B \succ A$ would imply $N_6 > 1/2$ (the blue-line area). Hence, in this case, we have $P =$*BAC*. When neither a population with preference *CAB* nor one with *BAC* wins a majority of votes, i.e., $N_3, N_6 < 1/2$, we will have either $A \succ C \succ B$ or $A \succ B \succ C$, depending on whether 5(b) holds or not. In either case, there is no CC because 5(a) always holds while 5(c) always fails.

To sum up, we have proved

Proposition 3: Suppose A1 and A2 hold. If $K-\alpha < \beta-\gamma$, then CC does not exist for any pattern of asset portfolios in the simplex $w_a + w_b + w_c = 1$.

A1 and A2 are merely technical assumptions that simplify the algebra. However, what does the condition $K-\alpha<\beta-\gamma$ mean? Why would a condition between price-change parameters dictate whether a CC exists? Here is our interpretation. That $K-\alpha$ is relatively small means that, for residents at site *c*, the price change parameter between site *a* and site *c* (that is, α) is not very different from the on-site price change parameter (K) at site *c*. Furthermore, $\beta-\gamma$ being relatively large means that, for residents at *b*, a park at site *a* is substantially more attractive than a park at site *c*. In view of Figure 1, the condition $K-\alpha<\beta-\gamma$ makes site *a* attractive for residents at both *b* and *c*. Thus, $K-\alpha<\beta-\gamma$ implies that, for all residents, the price changes *align* individual endowments to favor policy *A*.

In view of Figure 4, we also see that the possible individual preferences are *BAC*, *ABC*, *ACB* and *CAB*. Compared with the original six ordering possibilities, we find that the price-change condition $K-\alpha<\beta-\gamma$ rules out the possibility of *CBA* and *BCA*. With $K-\alpha<\beta-\gamma$, individuals will never prefer both policies *B* and *C* over *A*, simply because a park at site *a* will always generate an attractive increase in wealth for them.

4.4 The $\boldsymbol{K-\alpha>\beta-\gamma}$ Case

In Figure 5, we present the three hyperplanes for the benchmark case when $K>\alpha=\beta=\gamma$, which implies that $K-\alpha>\beta-\gamma$ holds. In this symmetric-parameter case, the three lines $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ intersect at the point (w_a, w_b)=(1/3,1/3). Of course, this means that w_c is also equal to 1/3. In this special symmetric case, all six triangles marked respectively by their population $(N_1, N_2, N_3, N_4, N_5, N_6)$, which correspond to the orderings *ABC*, *ACB*, *BCA*, *BAC*, *CAB* and *CBA,* respectively, are of equal area, 1/12. In contrast to Figure 4, in the case where $K>\alpha=\beta=\gamma$, individual orderings with *CBA* and *BCA* are both possible. As one can see from Figure 5, the set of asset portfolios corresponding to *CBA* and *BCA* refers to the case where individuals hold relatively fewer assets at site *a*, and they hold larger shares of assets at sites *b* and *c*.

To elaborate on the underlying implications, for the time being we make the following technical assumption:

Assumption 3 (A3): The asset portfolios $(\boldsymbol{w_a}, \boldsymbol{w_b}, \boldsymbol{w_c})$ satisfies Impartial Anonymous Culture assumption (all voting situations are equally likely) **at** $K>\alpha=\beta=\gamma$.[9]

In the benchmark case where $K>\alpha=\beta=\gamma$, given A3, since all triangles have the same area, we have voting ties for all kinds of preferences. By combining the geometric characteristics of the three hyperplanes outlined in Section 4.2, we can draw Figure 6 for the general case where $K>\alpha>\beta>\gamma$.

9 See, Gehrlein (2002)

For any $K > \alpha > \beta > \gamma$ case, we imagine that it is "transformed" from the original $K > \alpha = \beta = \gamma$ case in Figure 5.[10] Because (1/2, 1/2), (0, 1/2), and (1/2, 0) are the three points which must be passed through by the lines $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$, respectively,[11] the transformation from the case $\alpha = \beta = \gamma$ in Figure 5 to the case $\alpha > \beta > \gamma$ in Figure 6 can be seen as a rotation of the three lines $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$. Given the fixed point that has to be passed through, it is easy to verify the direction of rotation by checking the change in the slope of each line. When we change from $\alpha = \beta = \gamma$ to $\alpha > \beta > \gamma$, for line $\overline{AB}$, it is a clockwise rotation, fixing the point (w_a, w_b)=(1/2, 1/2). For line $\overline{BC}$, it is a counter-clockwise rotation, fixing the point (w_a, w_b)=(0, 1/2); and for line $\overline{CA}$, it is also a counter-clockwise rotation, fixing the point (w_a, w_b)=(1/2, 0).

When we rotate the partition lines as described above, we can see the changes in the partitioned areas. The rotations of line $\overline{AB}$ and line $\overline{CA}$ after the price-change vector moves from $K > \alpha = \beta = \gamma$ to $K > \alpha > \beta > \gamma$ make the areas of the asset endowments for individuals who prefer *A* to *B* and *A* to *C* both become larger. Hence, given A3, if before the rotation there are ties, then after the rotation pairwise majority voting must imply that $A \succ B$ and $A \succ C$. Therefore, there is no CC in the $\alpha > \beta > \gamma$ case, and policy *A* is the Condorcet winner. Thus, we have

Proposition 4: Suppose A1, A2 and A3 hold. If $K > \alpha > \beta > \gamma$, then CC does not exist and *A* is the Condorcet winner.

Note that assumption A3 is actually sufficient but not necessary to sustain Proposition 4. Even if the endowment portfolio is not uniformly distributed, the above analysis still shows that the proportion of population supporting both $A \succ B$ and $A \succ C$ will increase after the rotation, thereby increasing the probability of the event that *A* is the Condorcet winner.

4.5 Condorcet Winner and Development Values

We have shown in the previous discussion how introducing the property market into a society helps to avoid CC and restore the transitivity of the majority-rule mechanism. In this capitalist society, because all voters prefer policies that increase their total wealth more, the next question would be: Will the democratic majority rule end up choosing a wealth-maximizing development plan?

Let M_a, M_b and M_c be the total monetary value of properties at sites *a, b* and *c*,

---

[10] One can imagine that $\beta = \gamma$ first departs from α, and becomes less than α. Then γ departs from β and becomes less than β.

[11] Recall the facts of these hyperplanes summarized in Subsection 4.2.

respectively. The total wealth increases from park development are as follows:

$$KM_a + \alpha M_c + \beta M_b \text{ for policy } A$$

$$KM_b + \beta M_a + \gamma M_c \text{ for policy } B$$

and

$$KM_c + \alpha M_a + \gamma M_b \text{ for policy } C.$$

Hence, a park at site a generates more monetary value than site b if and only if

$$M_a > M_b - \frac{\alpha-\gamma}{K-\beta} M_c.$$

Similarly, a park at site a generates more monetary value than site c if and only if

$$M_a > M_c - \frac{\beta-\gamma}{K-\alpha} M_c.$$

Note that by definition M_a, M_b and M_c are all positive numbers. Straightforward algebra leads us to the following proposition:

Proposition 5: Suppose A1 and A2 hold. If policy *A* turns out to be a Condorcet winner, it generates the highest property value increase if and only if[12]

$$M_a > \max\left\{M_b - \frac{\alpha-\gamma}{K-\beta} M_c, M_c - \frac{\beta-\gamma}{K-\alpha} M_b\right\}. \quad (6)$$

What does condition (6) mean? Note that in our model agents compare *percentage* changes in their wealth to order policies. It is therefore a bit farfetched to expect that the majority voting outcome turns out to be the one that maximizes the property *value*. We therefore consider the special case when the property values in the three sites before the park is built are not very different: $M_a \approx M_b \approx M_c$. In this case, condition (6) is always satisfied under the condition $K > \alpha > \beta > \gamma$. Hence, if the property values among the three sites are not very far apart, the Condorcet winner always selects the policy with the largest total wealth increase.

However, when the property value at site *a* is quite small, one can expect that the Condorcet winner based on percentage comparison may select an inefficient outcome. For example, let $K = 1, \alpha = 0.8, \beta = 0.5, \gamma = 0.4, M_a < 0.2M_b$, and $M_b = M_c$. In this case, the Condorcet winner selects the worst outcome in terms of the overall increase in the property value.

Nevertheless, when $K - \alpha < \beta - \gamma$, we have both $(\alpha-\gamma)/(K-\beta) > 1$ and $(\beta-\gamma)/(K-\alpha) > 1$. Hence, either $M_b - M_c(\alpha-\gamma)/(K-\beta) < 0$ or $M_c -$

12

We focus on the event that policy A is the Condorcet winner because this is more likely to happen when $K > \alpha > \beta > \gamma$. Note that policy *B* generates the highest property value increase if and only if $M_b > \max\{M_a + M_c(\alpha-\gamma)/(K-\beta), M_c + M_a(\alpha-\beta)/(K-\gamma)\}$, which is also less likely to happen than policy *A*.

$M_b(\beta-\gamma)/(K-\alpha) < 0$, or both. In this situation, therefore, no matter how small M_a is, the Condorcet winner is never the least efficient outcome among the three policies.

5. More General Discussion

The analysis in Section 3 demonstrates that when people order policy proposals based on the induced wealth changes, property prices play the "alignment" role. This effect in turn makes the domain of individual preferences "less universal" and hence reinstitutes the transitivity property of the majority voting mechanism. This section will elaborate on the above discussion, and point out its extensions and implications.

5.1 The Disturbance of Some "Nonaligned" Residents

Consider Figure 7, in which residents in *a, b,* and *c,* as in the previous discussion, have endowments (w_a, w_b, w_c) with $w_a + w_b + w_c = 1$. As shown in Propositions 3 and 4, CCs are not possible under reasonable assumptions. Without loss of generality, suppose that the majority voting applied to residents in *a, b* and *c* will constitute the social ordering $A \succ C \succ B$, and that $A \succ B$.

Now suppose residents at site *d* are somehow allowed to vote, but their property endowments are mainly (say, with a *1-q* proportion) in site-*d*, and only a relatively small proportion *q* is invested in *a, b* and *c* ($w_a + w_b + w_c = q$). A park developed at site *a, b* or *c* may also affect the property price at site *d*, which we denote in Figure 7 as $\delta_a, \delta_b, \text{and } \delta_c,$ respectively. When a park is developed at site *a,* for instance, the total wealth change for *d*-residents is

$$(w_a, w_b, w_c) \cdot (\Delta p_{aa}, \Delta p_{ab}, \Delta p_{ac}) + (1-q)\Delta p_{da} \equiv \Delta W_{ad},$$

where Δp_{da} is the percentage price-change at site *d* when a park is built at site *a*. Similarly, we can respectively write down the wealth change formulas for ΔW_{bd} and ΔW_{cd} when the park is developed at site *b* or site *c.* It is evident that when *q* is small, *d*-residents are only marginally aligned with the price-change vector $(\Delta p_{aa}, \Delta p_{ab}, \Delta p_{ac})$; they actually care more about δ_a instead.[13] In Figure 7, we draw $\delta_a > \delta_b > \delta_c$ to imply that $\Delta p_{dd} > \Delta p_{dc} > \Delta p_{db} > \Delta p_{da}$. Thus, *d*-residents are assumed to have the ordering *DCBA*. How would the introduction of these nonaligned *d*-residents affect the results?

Suppose the populations on sites *a, b* and *c* are partitioned into six areas, as shown in Figure 6. We normalize the total population in the six partitioned areas to

[13] One can imagine that *a, b* and *c* are three sites in California, and *d* is a site in Arizona. Arizona residents hardly mind the price changes in California.

become 1: $N_1+N_2+N_3+N_4+N_5+N_6=1$. We know that originally residents on sites *a, b* and *c* form a social ordering $A \succ C \succ B$, and $A \succ B$ also holds. In view of Figure 6, we see that this implies that the following inequalities must be true:

$$A \succ C: N_1+N_2+N_6 \equiv x > 0.5,$$
$$C \succ B: N_2+N_3+N_4 > 0.5,$$
$$A \succ B: N_1+N_2+N_3 \equiv z > 0.5.$$

We would like to ask the following question: When nonaligned *d*-residents are allowed to vote for *A*, *B*, and *C*, under what condition will the *d*-residents "disturb" the originally aligned ordering and form a CC?

Note that since d-residents have the ordering *DCBA*, their ordering between *B* and *C* is the same as that for residents in *a, b*, and *c* (they also order $C \succ \mathrm{B}$ by design). Thus, the only possible CC scenario is $A \succ C \succ B \succ A$. Suppose the population size at site *d* is ϕ. Given that $C \succ B$ is the common position of all residents, to have $A \succ C$ and $B \succ A$ (which together with $C \succ B$ form a CC), we must have

$$x > \frac{1+\phi}{2} > z. \quad (6)$$

For instance, if *x*=0.7 and *z*=0.6, then ϕ must be between 0.2 and 0.4.

Intuitively, ϕ cannot be too small; otherwise, it cannot reverse the original $A \succ B$ ordering among residents in *a, b* and *c* to form a CC. In addition, ϕ cannot be too large either, otherwise it will reverse $A \succ C$ into $C \succ A$, making *C* a Condorcet winner. To sum up, we have

Proposition 6: Suppose residents on sites *a, b,* and *c* have a social ordering $\boldsymbol{A \succ C \succ B}$, and that $\boldsymbol{A \succ B}$ also holds. Allowing *d*-residents to vote among *A*, *B* and *C* will reinstitute a Condorcet cycle if 1) *q* (*d*-residents' holdings of properties on sites *a, b* and *c*) is sufficiently small, and 2) inequality (6) holds.

5.2 The General Message

In the above discussion, we interpret δ_x's as the *d*-site price changes in response to a park built at site *x*. In reality, however, there are broader interpretations. In Arrow's (1963) term, $(\delta_a, \delta_b, \delta_c)$ may be a vector characterizing *d*-residents' *tastes* with respect to a park built on different sites. The δ's may be parameters characterizing residents' judgments of convenience (between site *d* and the park), or residents' subjective preferences where a park "should" be built. When the δ's are interpreted in this way, then *q* can be viewed as the weight that individuals attach to wealth, and 1-*q* is the weight attached to other taste factors.

Of course, a wealth-comparison is unlikely to be the only concern of individual ordering, and individual tastes do matter. The market price in a capitalist society can help to align endowments, but concerns other than changes in wealth can certainly broaden the domain of individual preferences. Because such concerns can hardly be aligned in a capitalist society, they broaden the preference domain, and thereby render a CC possible.

In practice, all central or local governments in democratic countries have some kinds of restrictions on voting eligibility. For instance, the Province of British Columbia in Canada requires that Canadian citizens "have either lived or *owned properties* in the jurisdiction in which they intend to vote for at least 30 days before they register to vote …".[14] This is indeed an "alignment" test, and property ownership is at least part of the common attachment. As we have shown in the text, one outcome of allowing nonaligned citizens to vote is the possibility of a CC. Property ownership as an eligibility criterion at least partly serves the role of eliminating nonaligned residents, and helps to form a consistent majority voting result.

6. Concluding Remarks

We have examined in this paper how private property affects individual decisions. One reasonable assumption in a capitalist society is that people would prefer policies that increase their property values. If wealth change is a major concern, we have shown that the price mechanism plays the role of aligning individual preferences. Leaving aside the idiosyncratic individual tastes, this alignment effect rules out the possibilities of a Condorcet Cycle, thereby causing the majority voting mechanism to fulfill the axiom of social-ordering transitivity.

Professor Arrow's renowned contributions to economics, among others, include the impossibility theorem that we have discussed in this paper and the first welfare theorem. The latter theorem states that all competitive equilibria must be Pareto-efficient. The insight behind the first welfare theorem is in fact not dissimilar to that described in this paper. Under a competitive equilibrium, every individual agent (say, a firm owner) faces the same market prices. If any agents do not equate their marginal rate of technical substitution to the ratio of factor prices, they forfeit the opportunity for efficiency improvement, and will thereby be driven out of the market. In this sense, the market prices also play the role of aligning all firms' marginal rates of technical substitution.

In this paper, we show that despite the possibly universal distribution of individual endowments, the market prices in a capitalist society somewhat align the preferences of individuals with different endowments. This alignment significantly

[14] See https://www2.gov.bc.ca/gov/content/governments/local-governments/governance-powers/general-local-elections/voter-eligibility-voting.

changes the original universality of individual orderings among policies, and results in individual choices having particular patterns. If individuals tend to choose policies that improve their total wealth, we show that the alignment effect of market prices may help eliminate the disturbing Condorcet cycles. However, whether the Condorcet winner maximizes the increase in total property values will depend on the distribution of the original wealth endowment in the society.

References

Arrow, Kenneth J. (1951) *Social Choice and Individual Values*, New York: John Wiley and Sons. (2nd Edition. 1963)

Brandal, Nikolai, Oivind Bratberg and Dag Einar Thorsen (2013) The Nordic Model of Social Democracy. N.Y.: Macmillan.

Condorcet, Marquis de (1785), Essai sur l'application de l'analyse à la probabilité des decisions rendues à la pluralité des voix, Paris.

DeMare, Brian James (2019) *Land Wars: The Story of China's Agrarian Revolution.* Palo Alto, CA: Stanford University Press.

DeMeyer, Frank & Plott, Charles R, 1970. "The probability of a cyclical majority," *Econometrica*, 38(2), 345-354.

Fitzpatrick, Sheila (1994) *Stalin's Peasants: Resistance and Survival in the Russian Village After Collectivization.* New York: Oxford University Press.

Gaertner, Wulf (1988) "Binary Inversions and Transitive Majority," in W. Eichhorn ed. *Measurement in Economics*, pp. 253-267. Heidelberg: Physica.

---- (2002) "Domain Restrictions," in K. J. Arrow, A. K. Sen and K. Suzumura eds. Handbook of Social Choice and Welfare, vol. 1, pp. 131-170. Amsterdam: North Holland.

Gehrlein, W.V. (2002) "Condorcet's paradox and the likelihood of its occurrence: different perspectives on balanced preferences," *Theory and Decision* **52,** 171–199.

Kenez, Peter (1999) *A History of the Soviet Union from Beginning to End*. Cambridge: Cambridge University Press.

Marx, Karl; David McLellan ed. (2008) *Capital: An Abridged Edition*. New York: Oxford University Press.

Maskin, Eric (1995) “Majority Rule, Social Welfare Functions, and Game Form,” in K. Basu, P. K. Pattanaik and K. Suzumura eds. Welfare and Development, Festschrift for Amartya Sen. Oxford: Clarendon Press.

McKelvey, R. D. (1976) “Intransitivities in multidimensional voting models and some implications for agenda control.” *Journal of Economic Theory*, 12(3), 472–482.

McKelvey, R. D. (1979) “General conditions for global intransitivities in formal voting models.” *Econometrica*, 47(5), 1085–1112.

Rawls, J. (1971). *A Theory of Justice.* Harvard University Press.

Riker, W. H. (1980) “Implications from the disequilibrium of majority rule for the study of institutions.” *American Political Science Review*, 74(2), 432–446.

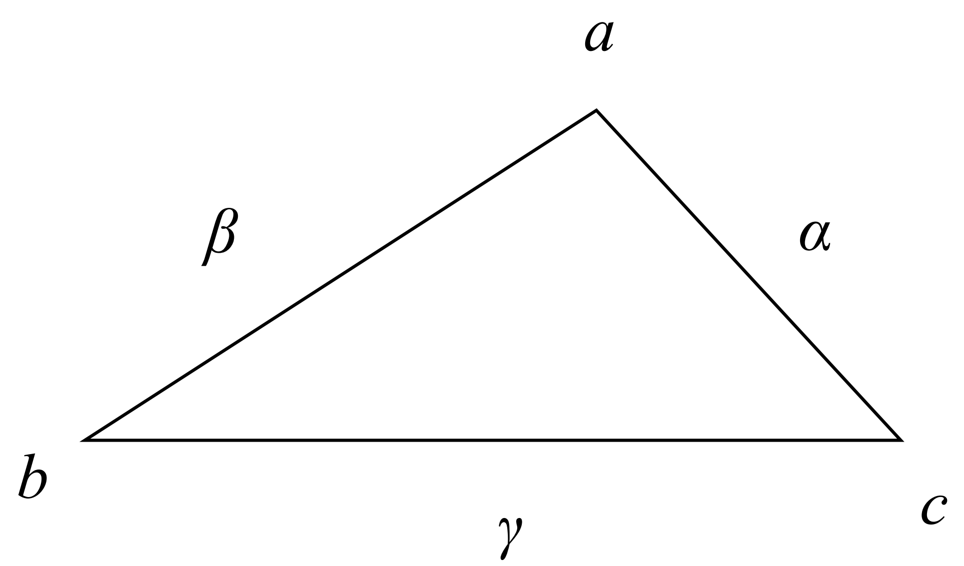
Schofield, N. (1978) “Instability of simple dynamic games.” *Review of Economic Studies*, 45(3), 575–594.

Sen, Amartya K. (1966) “A possibility theorem on majority decisions,” *Econometrica* 34: 491-499.

Sen, Amartya K. (1970) *Collective Choice and Social Welfare*. San Francisco: Holden-Day.

Sen, Amartya K. and Prasanta K. Pattanaik (1969) “Necessary and sufficient conditions for rational choice under majority decision,” *Journal of Economic Theory*, 1, 178-202.

Figure 1: The traveling convenience is $\overline{ac}$ better than $\overline{ab}$, and $\overline{ab}$ better than $\overline{bc}$. The price change parameters are $\alpha > \beta > \gamma$.



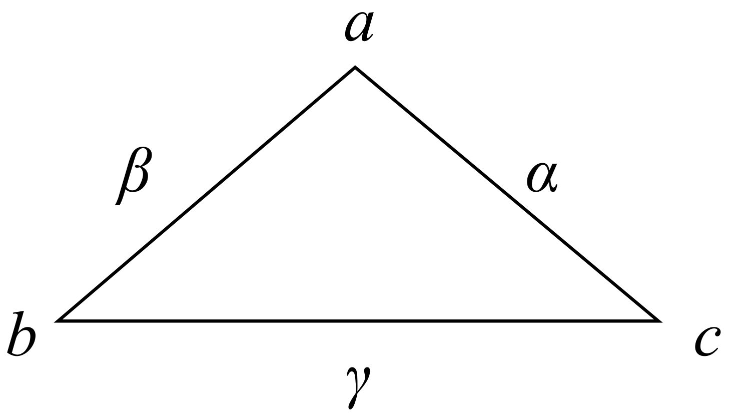


Figure 2a: $\alpha = \beta > \gamma$

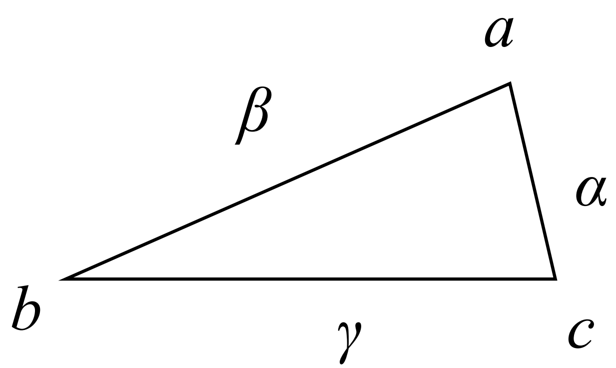


Figure 2b: $\alpha > \beta = \gamma$

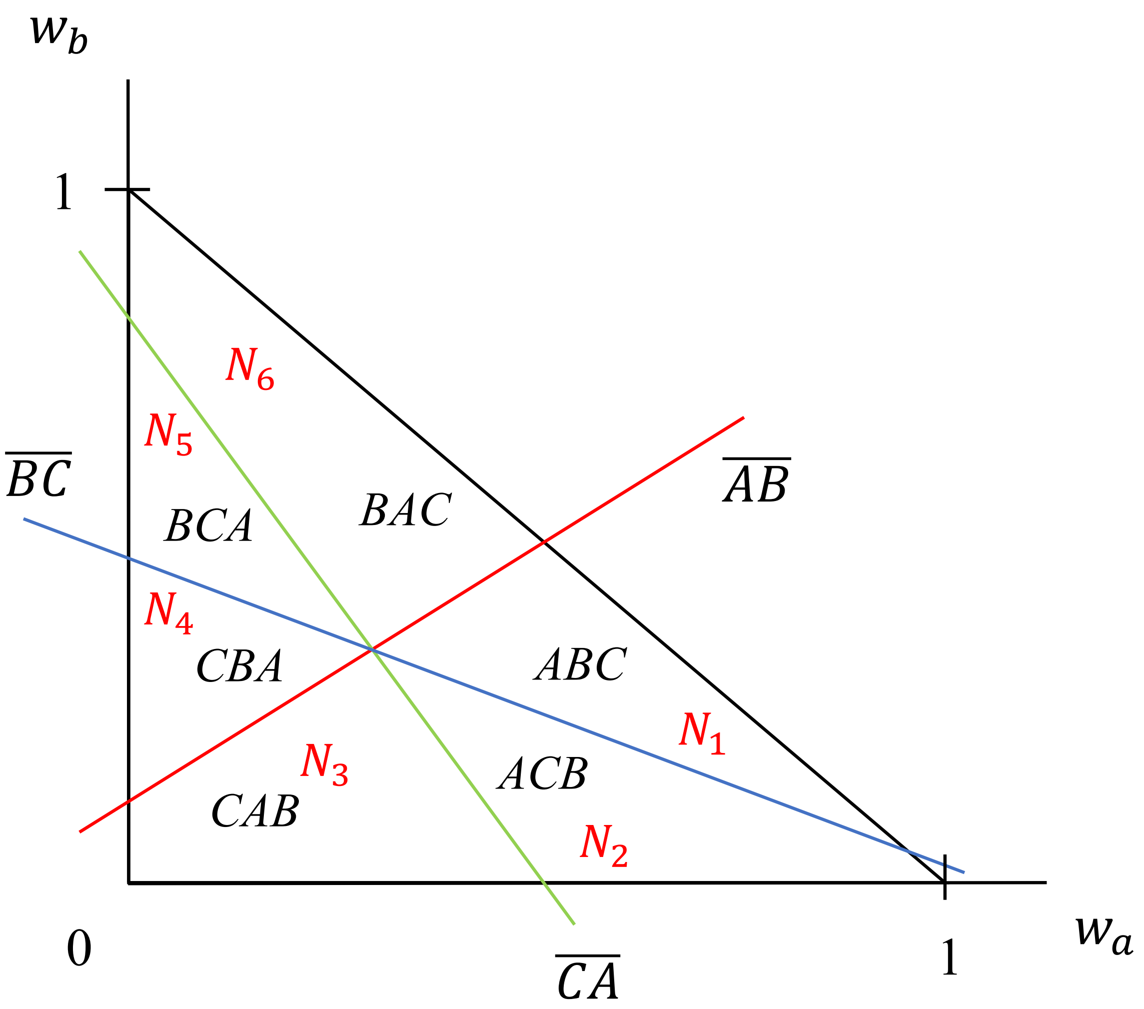


Figure 3: The six regions partitioned by the three lines $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

Figure 4: When K-$\alpha < \beta$-γ, the regions with $C\ B\ A$ and $B\ C\ A$ disappear

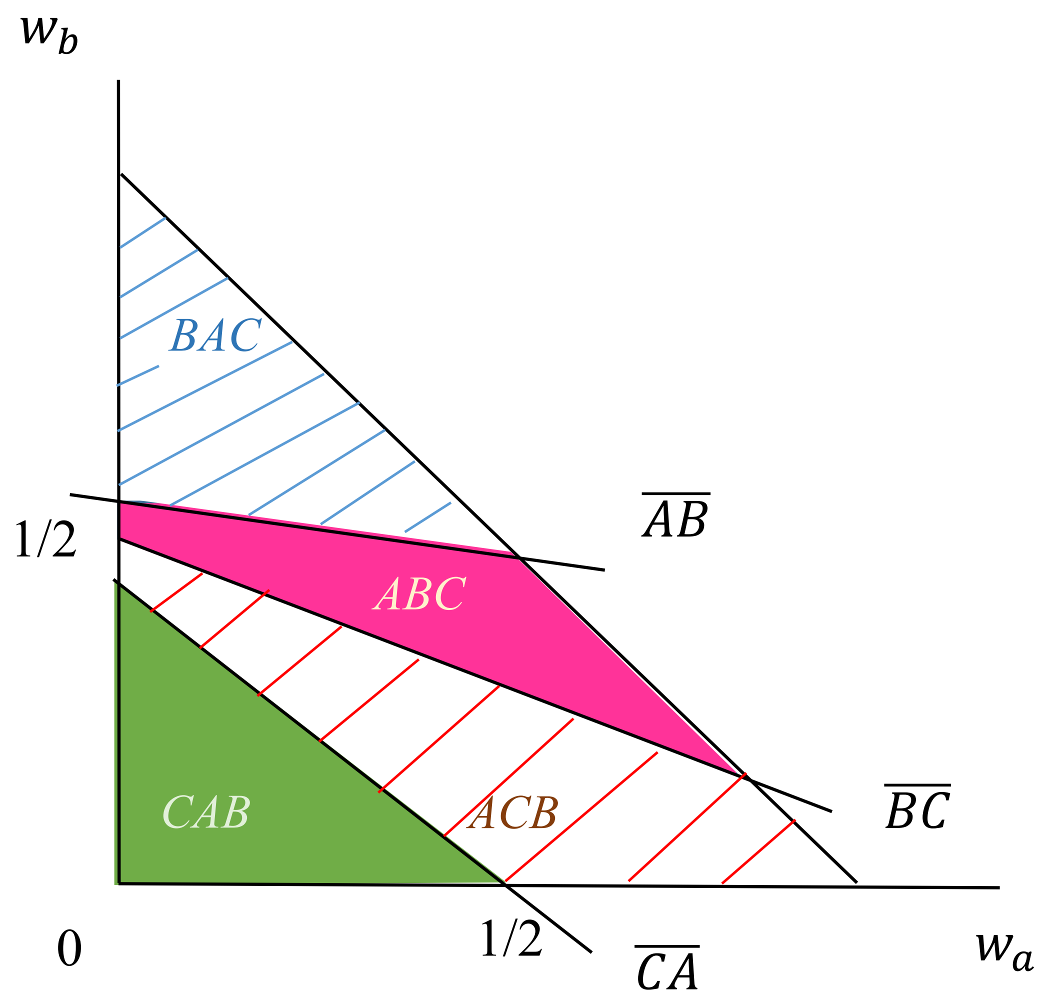


Figure 5: The partition when $\alpha = \beta = \gamma$. The six areas denoted $N_1, N_2, \dots, N_6$ all have the same area. If asset portfolios are uniformly distributed, we have ties: *A*~*B*, *B*~*C*, and *C*~*A*.

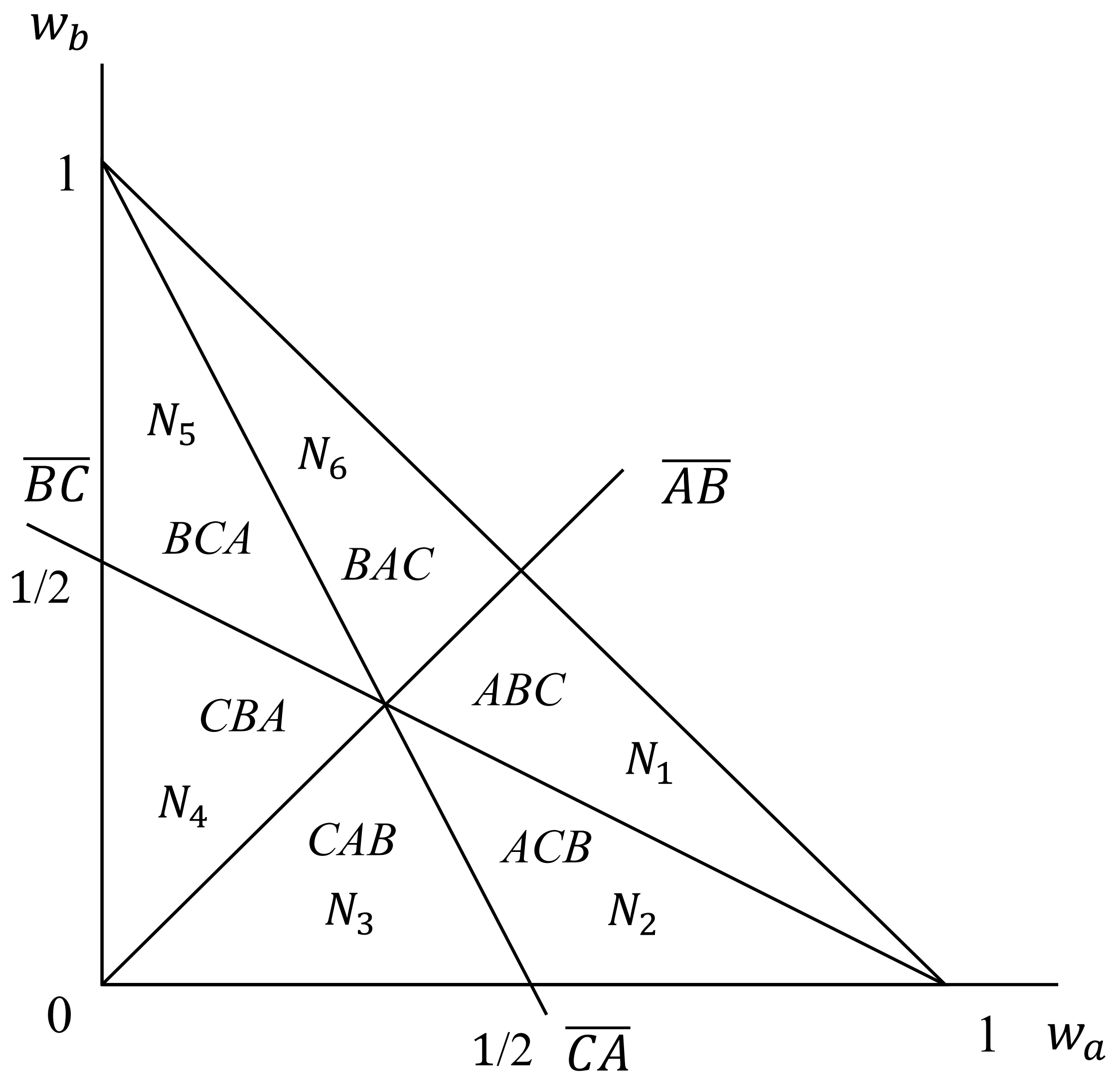


Figure 6: The partition when $\alpha > \beta > \gamma$. The case is "transformed" by rotating the original three lines in Figure 5 (now shown as blue dashed lines). We show in the text that various partitioned areas simultaneously change.

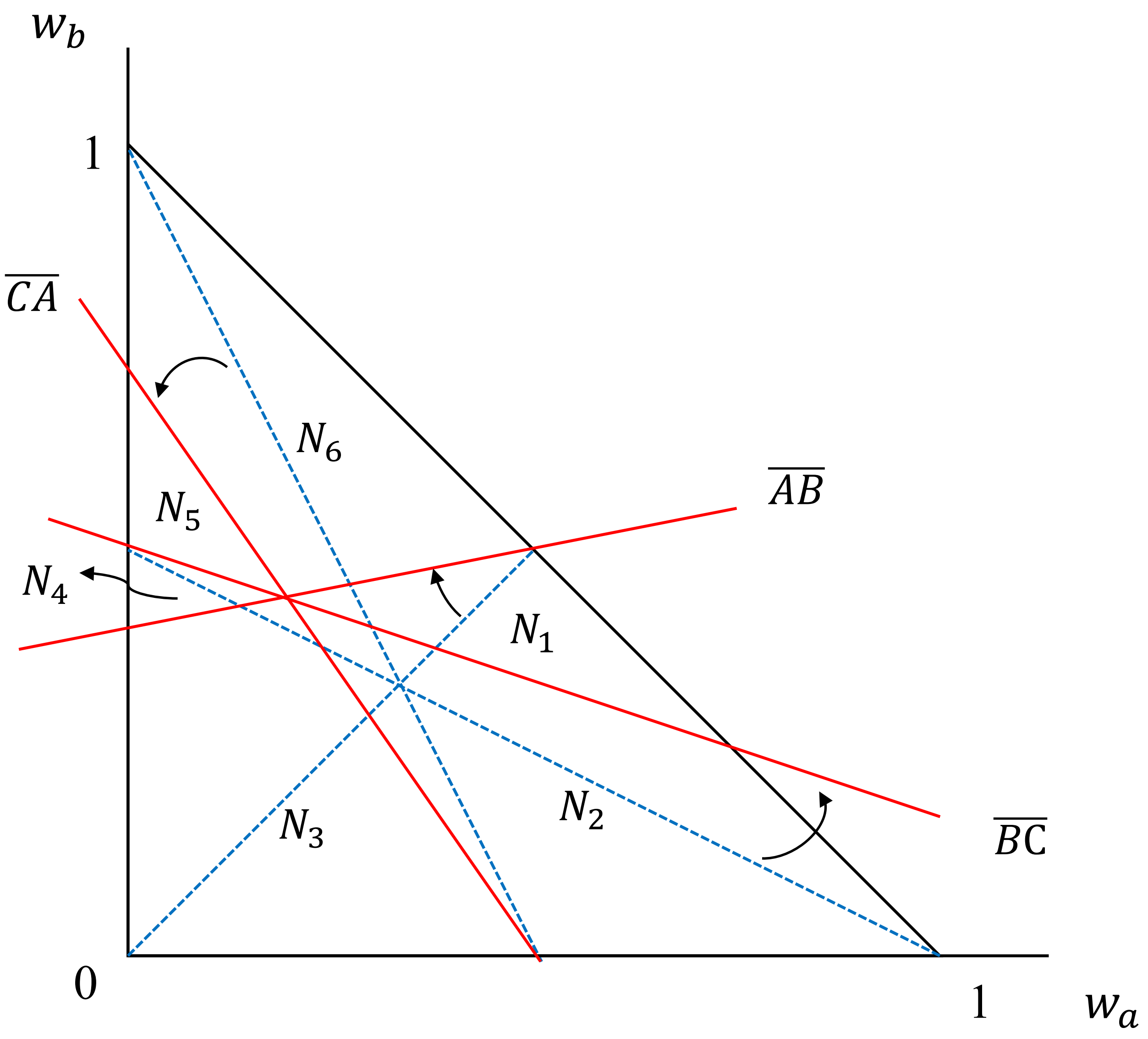


Figure 7: d-residents have concerns other than price changes in (a, b, c). These are represented by $(\delta_a, \delta_b, \delta_c)$. When d-residents' endowments in (a, b, c) are minor, those other concerns dominate, and we may have a CC $A \succ C \succ B \succ A$ in the four sites (a, b, c, d) situation.